

T.D.C. Part II, Chem. (Hons.)
Paper IV, Physical Chemistry (Gr. A)
Unit I Thermodynamics.

Entropy change in an ideal gas :-

Let one mole of an ideal gas undergo
occupying a volume V at a pressure P and temp T .
Let the δQ_{rev} amount be the amount of heat supplied to
the system reversibly, then the increase in entropy δS
is given by.

$$dS = \frac{\delta Q_{rev}}{T} \quad \text{or, } TdS = \delta Q_{rev} \quad \text{--- (1)}$$

From the first law of thermodynamics

$$\begin{aligned} \delta Q_{rev} &= dU + \delta W_{rev} \\ &= dU + PdV. \end{aligned} \quad \text{--- (2)}$$

From equation (1) and (2)

$$TdS = dU + PdV$$

$$\text{or, } dS = \frac{dU}{T} + \frac{PdV}{T}$$

$$\text{But } C_v = \frac{dU}{dT} \quad \text{or, } dU = C_v dT$$

$$\therefore dS = C_v \frac{dT}{T} + \frac{PdV}{T} \quad \text{--- (3)}$$

For one mole of an ideal gas

$$PV = RT \quad \text{or, } P = \frac{RT}{V}$$

Substituting this value P in equation (3) we get

$$dS = C_v \frac{dT}{T} + R \frac{dV}{V}$$

Integrating above equation by taking C_v as constant

$$S = C_v \ln T + R \ln V + S_0$$

Where S_0 = constant of integral and is called
Standard entropy.

The above equation gives entropy of one mole of an ideal gas at any P, V, T .

For the initial state

$$S_i = C_v \ln T_i + P \ln V_i + S_0$$

For the final state,

$$S_f = C_v \ln T_f + P \ln V_f + S_0$$

$$\therefore \Delta S = S_f - S_i = C_v \ln T_f + P \ln V_f + S_0 - C_v \ln T_i - P \ln V_i - S_0$$

or,

$$\Delta S = C_v \ln \frac{T_f}{T_i} + P \ln \frac{V_f}{V_i} \quad (4)$$

An alternative form of equation (4) may be obtained in the following manner;

$$C_p - C_v = R \quad \text{for a ideal gas}$$

So, $C_v = C_p - R$

$$S = C_p - R \ln T + P \ln \frac{RT}{P} + S_0$$

$$\begin{aligned} PV &= RT \\ \therefore V &= \frac{RT}{P} \end{aligned}$$

or,

$$S = C_p \ln T - P \ln T + P \ln RT - P \ln P + S_0$$

$$= C_p \ln T - P \ln T + P \ln R + R \ln T - P \ln P + S_0$$

$$S = q_p \ln T - p \ln p + (q_p \ln p + S_0)$$

$$= q_p \ln T - p \ln p + S_0'$$

For initial state

$$S_i = q_p \ln T_i - p \ln p_i$$

For final state

$$S_f = q_p \ln T_f - p \ln p_f$$

$$\Delta S = S_f - S_i = q_p \ln T_f - p \ln p_f - q_p \ln T_i + p \ln p_i$$

$$\Delta S = q_p \ln \frac{T_f}{T_i} - p \ln \frac{p_f}{p_i}$$

Equations (4) and (5) used for calculating entropy change for ideal or perfect gases.

(1) At constant pressure i.e.

$$p_i = p_f$$

$$\Delta S_p = q_p \ln \frac{T_f}{T_i}$$

(2) At constant volume i.e.

$$V_i = V_f$$

$$\Delta S_v = c_v \ln \frac{T_f}{T_i}$$

(13)

At constant temp

$$T_1 = T_2$$

$$\Delta S_T = p \ln \frac{V_2}{V_1}$$

$$\text{or, } \Delta S_T = -p \ln \frac{P_2}{P_1}$$